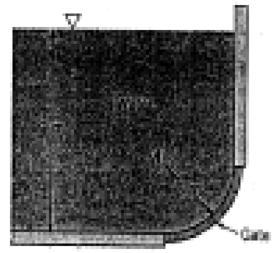
HW4 Solution

Group Homework Problem

2.70

2.70 A 4-m-long curved gate is located in the side of a reservoir containing water as shown in Fig. P2.70. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate. Will this force pass through point A? Explain.



■ FIGURE P2.70

For equilibrium,

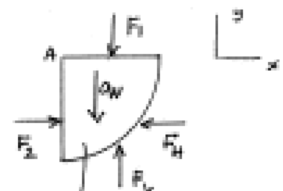
$$\sum F_x = 0$$

or

$$F_H = F_2 = \gamma h_{c2} A_2 = \gamma (6\,m + 1.5\,m)(3\,m \times 4\,m)$$

so that

$$F_H = \left(9.80 \frac{kN}{m^3}\right)(7.5\,m)(12\,m^2) = \underline{882\,kN}$$



volume ~ $\forall$

and

$$\forall = \frac{\pi (3\,m)^2}{4} \times 4\,m = 9\pi\,m^3$$

Similarly,

$$\sum F_y = 0$$

$$F_V = F_1 + \mathcal{W} \quad \text{where:}$$

$$F_1 = [\gamma(6\,m)](3\,m \times 4\,m) = \left(9.80 \frac{kN}{m^3}\right)(6\,m)(12\,m^2)$$

$$\mathcal{W} = \gamma \forall = \left(9.80 \frac{kN}{m^3}\right)(9\pi\,m^3)$$

Thus,

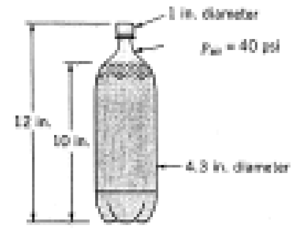
$$F_V = \left(9.80 \frac{kN}{m^3}\right)\left[72\,m^3 + 9\pi\,m^3\right] = \underline{983\,kN}$$

(Note: Force of water on gate will be opposite in direction to that shown on figure.)

The direction of all differential forces acting on the curved surface is perpendicular to surface, and therefore, the resultant must pass through the intersection of all these forces which is at point A. Yes.

2.71

2.71 The air pressure in the top of the two liter pop bottle shown in Video V2.4 and Fig. P2.71 is 40 psi, and the pop depth is 10 in. The bottom of the bottle has an irregular shape with a diameter of 4.3 in. (a) If the bottle cap has a diameter of 1 in. what is magnitude of the axial force required to hold the cap in place? (b) Determine the force needed to secure the bottom 2 inches of the bottle to its cylindrical sides. For this calculation assume the effect of the weight of the pop is negligible. (c) By how much does the weight of the pop increase the pressure 2 inches above the bottom? Assume the pop has the same specific weight as that of water.



■ FIGURE P2.71

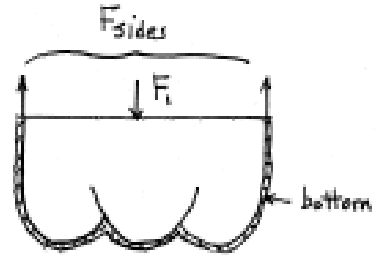
(a) $F_{cap} = p_{air} \times Area_{cap} = \left(40 \frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(1 \text{ in.})^2 = \underline{\underline{31.4 \text{ lb}}}$

(b) $\sum F_{vertical} = 0$

$F_{sides} = F_1 = $ (pressure @ 2 in. above bottom) $\times$ (Area)

$= \left(40 \frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(4.3 \text{ in.})^2$

$= \underline{\underline{581 \text{ lb}}}$



(c) $p = p_{air} + \gamma h$

$= 40 \frac{lb}{in.^2} + \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{8}{12} \text{ ft}\right)\left(\frac{1}{144 \text{ in.}^2/\text{ft}^2}\right)$

$= 40 \frac{lb}{in.^2} + 0.289 \frac{lb}{in.^2}$

Thus, the increase in pressure due to weight = $\underline{\underline{0.289 \text{ psi}}}$ (which is less than 1% of air pressure).

2.73

2.73 A plug in the bottom of a pressurized tank is conical in shape as shown in Fig. P2.73. The air pressure is 40 kPa and the liquid in the tank has a specific weight of 27 kN/m³. Determine the magnitude, direction, and line of action of the force exerted on the curved surface of the cone within the tank due to the 40 kPa pressure and the liquid.

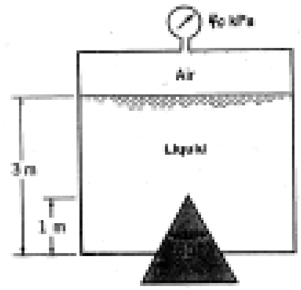
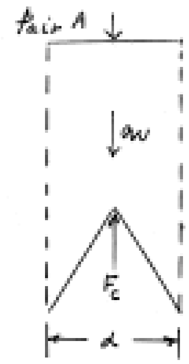


FIGURE P2.73



$\tan 30° = \dfrac{\frac{d}{2}}{1}$

$d = 2 \tan 30° = 1.155\ m$

volume of cone $= \frac{\pi}{3}\left(\frac{d}{2}\right)^2(1)$

For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$F_c = p_{air} A + \mathcal{W}$$

where F_c is the force the cone exerts of the fluid.

Also,

$$p_{air} A = (40\ kPa)\left(\frac{\pi}{4}\right)(d^2)$$

$$= (40\ kPa)\left(\frac{\pi}{4}\right)(1.155\ m)^2 = 41.9\ kN$$

and

$$\mathcal{W} = \gamma\left[\frac{\pi}{4} d^2 (3\ m) - \frac{\pi}{3}\left(\frac{d}{2}\right)^2 (1\ m)\right]$$

$$= \gamma \pi d^2 \left[\frac{3\ m}{4} - \frac{1\ m}{12}\right]$$

$$= \left(27 \frac{kN}{m^3}\right)(\pi)(1.155\ m)^2\left(\frac{2}{3}\ m\right) = 75.4\ kN$$

Thus,

$$F_c = 41.9\ kN + 75.4\ kN = 117\ kN$$

and the force on the cone has a magnitude of 117 kN and is directed vertically downward along the cone axis.

2.75

2.75 The concrete (specific weight = 150 lb/ft³) seawall of Fig. P2.75 has a curved surface and restrains seawater at a depth of 24 ft. The trace of the surface is a parabola as illustrated. Determine the moment of the fluid force (per unit length) with respect to an axis through the toe (point A).

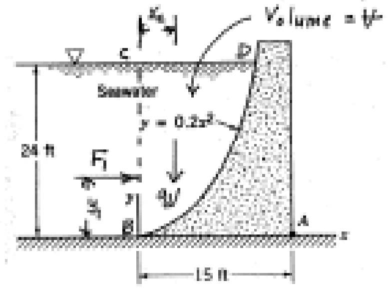


FIGURE P2.75

The components of the fluid force acting on the wall are F_1 and $\mathcal{W}$ as shown on the figure where

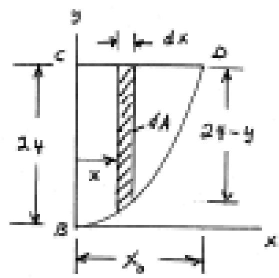
$$F_1 = \gamma h_c A = \left(64.0 \frac{lb}{ft^3}\right)\left(\frac{24\,ft}{2}\right)(24\,ft \times 1\,ft)$$

$$= 18{,}400\ lb \qquad \text{and} \qquad y_1 = \frac{24\,ft}{3} = 8\,ft$$

Also,

$$\mathcal{W} = \gamma ⩝$$

To determine ⩝ find area BCD. Thus, (see figure to right)



$x_o = \sqrt{120}$

(Note: All lengths in ft)

$$A = \int_0^{x_o} (24 - y)\,dx = \int_0^{x_o} (24 - 0.2x^2)\,dx$$

$$= \left[24x - \frac{0.2x^3}{3}\right]_0^{x_o}$$

and with $x_o = \sqrt{120}$, $A = 175\ ft^2$ so that

$$⩝ = A \times 1\,ft = 175\ ft^3$$

Thus,

$$\mathcal{W} = \left(64.0 \frac{lb}{ft^3}\right)(175\ ft^3) = 11{,}200\ lb$$

To locate centroid of A:

$$x_c A = \int_0^{x_o} x\,dA = \int_0^{x_o} (24 - y)\,x\,dx = \int_0^{x_o} (24x - 0.2x^3)\,dx = 12x_o^2 - \frac{0.2x_o^4}{4}$$

and

$$x_c = \frac{12(\sqrt{120})^2 - \frac{0.2(\sqrt{120})^4}{4}}{175} = 4.11\ ft$$

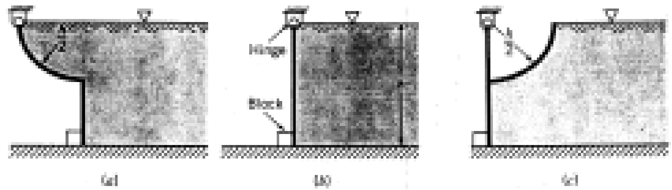
Thus,

$$M_A = F_1 y_1 - \mathcal{W}(15 - x_c)$$

$$= (18{,}400\ lb)(8\,ft) - (11{,}200\ lb)(15\,ft - 4.11\,ft) = \underline{25{,}200\ ft \cdot lb}$$

2.81

2.81 Three gates of negligible weight are used to hold back water in a channel of width b as shown in Fig. P2.81. The force of the gate against the block for gate (b) is R. Determine (in terms of R) the force against the blocks for the other two gates.

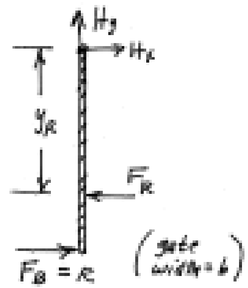


■ FIGURE P2.81

For case (b)

$$F_R = \gamma h_c A = \gamma\left(\frac{h}{2}\right)(h \times b) = \frac{\gamma h^2 b}{2}$$

and $y_R = \frac{2}{3}h$



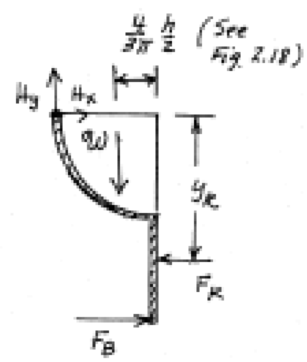
Thus,

so that $\Sigma M_H = 0$

$$hR = \left(\frac{2}{3}h\right)F_R$$

$$hR = \left(\frac{2}{3}h\right)\left(\frac{\gamma h^2 b}{2}\right)$$

$$R = \frac{\gamma h^2 b}{3} \qquad (1)$$



For case (a) on free-body-diagram shown

$F_R = \frac{\gamma h^2 b}{2}$ (from above) and

$y_R = \frac{2}{3}h$

and

$$\mathcal{W} = \gamma \times \mathcal{V}ol$$

$$= \gamma\left[\frac{\pi\left(\frac{h}{2}\right)^2}{4}(b)\right]$$

$$= \frac{\pi\gamma h^2 b}{16}$$

Thus, $\Sigma M_H = 0$

so that

$$\mathcal{W}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + F_R\left(\frac{2}{3}h\right) = F_B\, h$$

and

$$\frac{\pi\gamma h^2 b}{16}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + \frac{\gamma h^2 b}{2}\left(\frac{2}{3}h\right) = F_B\, h$$

(Con't)

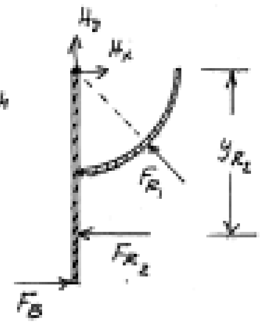
2.81 (Con't)

It follows that

$$F_B = \gamma h^2 b\,(0.390)$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \underline{\underline{1.17R}}$$



For case (c), for the free-body-diagram shown, the force F_{R_1} on the curved section passes through the hinge and therefore does not contribute to the moment around H. On bottom part of gate

$$F_{R_2} = \gamma h_c A = \gamma\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right) = \frac{3}{8}\gamma h^2 b$$

and

$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(b)\left(\frac{h}{2}\right)^3}{\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right)} + \frac{3h}{4}$$

$$= \frac{28}{36}h$$

Thus,

$$\Sigma M_H = 0$$

so that

$$F_{R_2}\left(\frac{28}{36}h\right) = F_B\, h$$

or

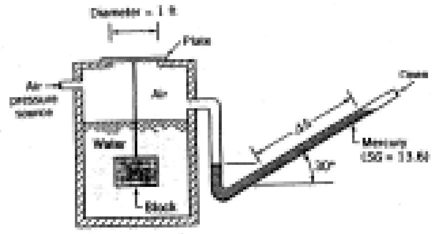
$$F_B = \left(\frac{3}{8}\gamma h^2 b\right)\left(\frac{28}{36}\right) = \frac{7}{24}\gamma h^2 b$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \frac{7}{8}R = \underline{\underline{0.875R}}$$

2.88

2.88 A plate of negligible weight closes a 1-ft diameter hole in a tank containing air and water as shown in Fig. P2.88. A block of concrete (specific weight = 150 lb/ft³), having a volume of 1.5 ft³, is suspended from the plate and is completely immersed in the water. As the air pressure is increased the differential reading, Δh, on the inclined-tube mercury manometer increases. Determine Δh just before the plate starts to lift off the hole. The weight of the air has a negligible effect on the manometer reading.



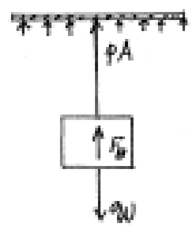
■ FIGURE P2.33

For equilibrium,

$$\Sigma F_{vertical} = 0$$

So that

$$\mathcal{W} = pA + F_B$$



where:

$\mathcal{W}$ ~ weight of concrete
p ~ air pressure
A ~ area of plate
F_B ~ b

Thus,

$$\left(150\,\frac{lb}{ft^3}\right)(1.5\,ft^3) = p\left(\frac{\pi}{4}\right)(1\,ft)^2 + \left(62.4\,\frac{lb}{ft^3}\right)(1.5\,ft^3)$$

so that

$$p = 167\,\frac{lb}{ft^2}$$

The manometer equation is

$$p = \gamma_{Hg}\,\Delta h\,\sin 30°$$

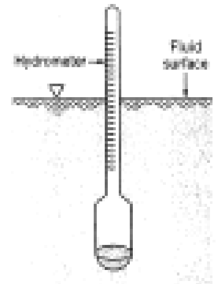
so that

$$\Delta h = \frac{p}{\gamma_{Hg}\sin 30°}$$

$$= \frac{167\,\frac{lb}{ft^2}}{\left(847\,\frac{lb}{ft^3}\right)\sin 30°} = \underline{\underline{0.394\,ft}}$$

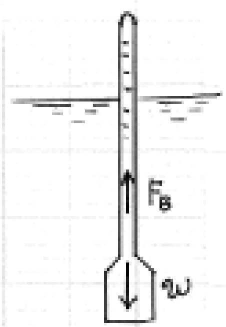
2.89

2.89 When a hydrometer (see Fig. P2.89 and Video V2.6) having a stem diameter of 0.30 in. is placed in water, the stem protrudes 3.15 in. above the water surface. If the water is replaced with a liquid having a specific gravity of 1.10, how much of the stem would protrude above the liquid surface? The hydrometer weighs 0.042 lb.



■ FIGURE P2.89

When the hydrometer is floating its weight, $\mathcal{W}$, is balanced by the buoyant force, F_B. For equilibrium,



$$\sum F_{vertical} = 0$$

Thus, for water

$$F_B = \mathcal{W}$$

$$(\gamma_{H_2O}) \forall_1 = \mathcal{W} \qquad (1)$$

where $\forall_1$ is the submerged volume. With the new liquid

$$(SG)(\gamma_{H_2O}) \forall_2 = \mathcal{W} \qquad (2)$$

Combining Eqs. (1) and (2) with $\mathcal{W}$ constant

$$(\gamma_{H_2O}) \forall_1 = (SG)(\gamma_{H_2O}) \forall_2$$

and

$$\forall_2 = \frac{\forall_1}{SG} \qquad (3)$$

(con't)

2.89 (Con't)

From Eq. (1)

$$\forall_1 = \frac{\mathcal{W}}{\gamma_{H_2O}} = \frac{0.042 \text{ lb}}{62.4 \frac{\text{lb}}{\text{ft}^3}} = 6.73 \times 10^{-4} \text{ ft}^3$$

so that from Eq. (3)

$$\forall_2 = \frac{6.73 \times 10^{-4} \text{ ft}^3}{1.10} = 6.12 \times 10^{-4} \text{ ft}^3$$

Thus, $\forall_1 - \forall_2 = (6.73 - 6.12) \times 10^{-4} \text{ ft}^3 = 0.61 \times 10^{-4} \text{ ft}^3$

To obtain this difference the change in length, $\Delta \ell$, is

$$\left(\frac{\pi}{4}\right)(0.30 \text{ in.})^2 \Delta \ell = (0.61 \times 10^{-4} \text{ ft}^3)\left(1728 \frac{\text{in.}^3}{\text{ft}^3}\right)$$

$$\Delta \ell = 1.49 \text{ in.}$$

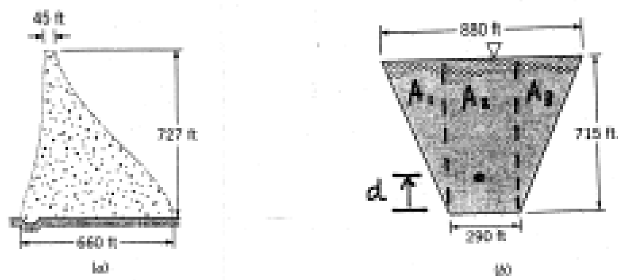
With the new liquid the stem would protrude

$3.15 \text{ in.} + 1.49 \text{ in.} = \underline{\underline{4.64 \text{ in.}}}$ above the surface

Individual Homework Problem

2-72

2.72 Hoover Dam (see Video 2.3) is the highest arch-gravity type of dam in the United States. A cross section of the dam is shown in Fig. P2.72(a). The walls of the canyon in which the dam is located are sloped, and just upstream of the dam the vertical plane shown in Figure P2.72(b) approximately represents the cross section of the water acting on the dam. Use this vertical cross section to estimate the resultant horizontal force of the water on the dam, and show where this force acts.



■ FIGURE P2.72

Break area into 3 parts as shown.

For area 1:

$F_{R_1} = \gamma h_c A_1 = (62.4 \frac{lb}{ft^3})(\frac{1}{3})(715\ ft)(\frac{1}{2})(295\ ft)(715\ ft)$

$= 1.57 \times 10^9\ lb$

For area 3: $F_{R_3} = F_{R_1} = 1.57 \times 10^9\ lb$

For area 2:

$F_{R_2} = \gamma h_c A_2 = (62.4 \frac{lb}{ft^3})(\frac{1}{2})(715\ ft)(290\ ft)(715\ ft)$

$= 4.63 \times 10^9\ lb$

Thus,

$F_R = F_{R_1} + F_{R_2} + F_{R_3} = 1.57 \times 10^9\ lb + 4.63 \times 10^9\ lb + 1.57 \times 10^9\ lb$

$= 7.77 \times 10^9\ lb$

Since the moment of the resultant force about the base of the dam must be equal to the moments due to F_{R_1}, F_{R_2}, and F_{R_3}, it follows that

(con't)

2.72 (con't)

$$F_R \times d = F_{R_1}\left(\frac{2}{3}\right)(715\text{ ft}) + F_{R_2}\left(\frac{1}{2}\right)(715\text{ ft}) + F_{R_3}\left(\frac{2}{3}\right)(715\text{ ft})$$

and

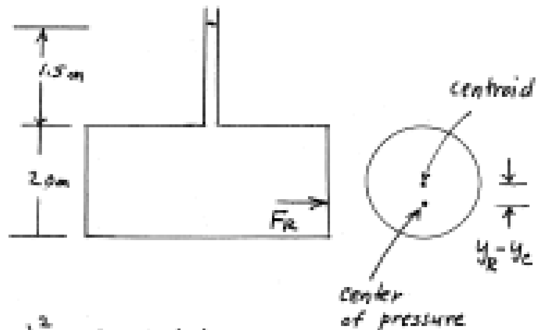
$$d = \frac{(1.57\times10^9\text{ lb})\left(\frac{2}{3}\right)(715\text{ ft}) + (4.63\times10^9\text{ lb})\left(\frac{1}{2}\right)(715\text{ ft}) + (1.57\times10^9\text{ lb})\left(\frac{2}{3}\right)(715\text{ ft})}{7.77\times10^9\text{ lb}}$$

$$= 406\text{ ft}$$

Thus, the resultant horizontal force on the dam is <u>7.77×10^9 lb acting 406 ft up from the base of the dam along the axis of symmetry of the area.</u>

2.76

2.76 A cylindrical tank with its axis horizontal has a diameter of 2.0 m and a length of 4.0 m. The ends of the tank are vertical planes. A vertical, 0.1-m-diameter pipe is connected to the top of the tank. The tank and the pipe are filled with ethyl alcohol to a level of 1.5 m above the top of the tank. Determine the resultant force of the alcohol on one end of the tank and show where it acts.



$F_R = \gamma h_c A$

where $h_c = 1.5m + 1.0m = 2.5m$

so that

$$F_R = \left(7.74 \frac{kN}{m^3}\right)(2.5\,m)\left(\frac{\pi}{4}\right)(2.0\,m)^2 = 60.8\,kN$$

Also,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

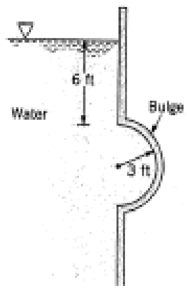
where $y_c = h_c$ so that

$$y_R = \frac{\frac{\pi (1m)^4}{4}}{(2.5m)\left(\frac{\pi}{4}\right)(2m)^2} + 2.5\,m = 2.60\,m$$

Thus, the resultant force has a magnitude of <u>60.8 kN</u> and acts at a distance of $y_R - y_c = 2.60m - 2.50m =$ <u>0.100m below center of tank end wall</u>.

2.78

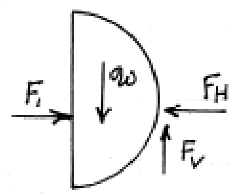
2.78 An open tank containing water has a bulge in its vertical side that is semicircular in shape as shown in Fig. P2.78. Determine the horizontal and vertical components of the force that the water exerts on the bulge. Base your analysis on a 1-ft length of the bulge.



■ FIGURE P2.78

F_H ~ horizontal force of wall on fluid

F_V ~ vertical force of wall on fluid



$$\mathcal{W} = \gamma_{H_2O} \forall_{ol}$$
$$= \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{\pi (3 ft)^2}{2}\right)(1 \ ft)$$
$$= 882 \ lb$$

$$F_1 = \gamma h_c A = \left(62.4 \frac{lb}{ft^3}\right)(6 ft + 3 ft)(6 ft \times 1 ft)$$
$$= 3371 b$$

For equilibrium, $F_V = \mathcal{W} = 882 \ lb \uparrow$
and $F_H = F_1 = 337 \ lb \leftarrow$

The force the water exerts on the bulge is equal to, but opposite in direction to F_V and F_H above. Thus,

$$\underline{(F_H)_{wall} = 337 \ lb \rightarrow}$$
$$\underline{(F_V)_{wall} = 882 \ lb \downarrow}$$

2.80

2.80 If the bottom of a pop bottle similar to that shown in Fig. P2.71 and in Video V2.4 were changed so that it was hemispherical, as in Fig. P2.80, what would be the magnitude, line of action, and direction of the resultant force acting on the hemispherical bottom? The air pressure in the top of the bottle is 40 psi, and the pop has approximately the same specific gravity as that of water. Assume that the volume of pop remains at 2 liters.

FIGURE P2.80

Force = weight of pop supported by bottom + force due to air pressure

Weight of pop $= \gamma_{pop} \times$ volume of pop (1)

Volume = 2 liters $= (2\times10^{-3}\ m^3)\times\left(3.531\times10\ \frac{ft^3}{m^3}\right) = 0.0706\ ft^3$

Thus, from Eq. (1)

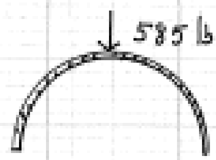
Weight of pop $= \left(62.4\ \frac{lb}{ft^3}\right)(0.0706\ ft^3) = 4.41\ lb$

Force due to air pressure $= p_{air} \times$ projected area of hemispherical bottom

$= \left(40\ \frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(4.3\ in.)^2$

$= 581\ lb$

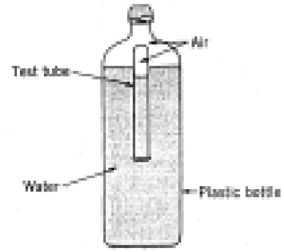
Resultant force $= 4.41\ lb + 581\ lb = \underline{585\ lb}$



The resultant force is directed vertically downward, and due to symmetry, it acts on the hemispherical bottom along the vertical axis of the bottle.

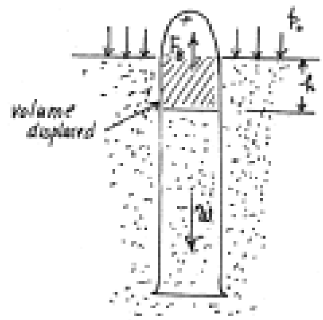
2.86

2.86 An inverted test tube partially filled with air floats in a plastic water filled soft drink bottle as shown in Video V2.5 and Fig. P2.86. The amount of air in the tube has been adjusted so that it just floats. The bottle cap is securely fastened. A slight squeezing of the plastic bottle will cause the test tube to sink to the bottom of the bottle. Explain this phenomenon.



■ FIGURE P2.86

When the test tube is floating the weight of the tube, $\mathcal{W}$, is balanced by the buoyant force, F_B, as shown in the figure. The buoyant force is due to the displaced volume of water as shown. This displaced volume is due to the air pressure, p, trapped in the tube where $p = p_0 + \gamma_{H_2O} h$. When the bottle is squeezed, the air pressure in the bottle, p_0, is increased slightly and this in turn increases p, the pressure compressing the air in the test tube. Thus, the displaced volume is decreased with a subsequent decrease in F_B. Since $\mathcal{W}$ is constant, a decrease in F_B will cause the test tube to sink.



2.90

2.90 The thin-walled, 1-m-diameter tank of Fig. P2.90 is closed at one end and has a mass of 90 kg. The open end of the tank is lowered into the water and held in the position shown by a steel block having a density of 7840 kg/m³. Assume that the air that is trapped in the tank is compressed at a constant temperature. Determine: **(a)** the reading on the pressure gage at the top of the tank, and **(b)** the volume of the steel block.

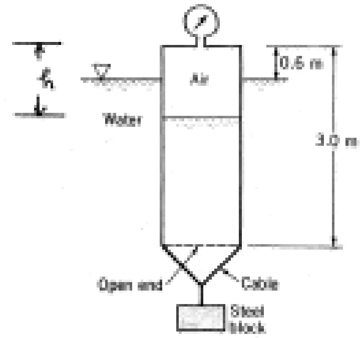


FIGURE P2.90

(a) For constant temperature compression,

$$p_i \forall_i = p_f \forall_f \quad \text{where } i \sim \text{initial state and } f \sim \text{final state.}$$

Let $\forall_f = A_t h$ (see figure) where A_t is the cross sectional area of tank, and

$$p_f = \gamma(h - 0.6) + p_{atm} \quad \text{(where all lengths are in m).} \qquad (1)$$

Thus,

$$\forall_f = A_t h = \frac{p_i \forall_i}{p_f}$$

Since $p_i = p_{atm}$ and $\forall_i = A_t(3)$

$$h = \frac{p_{atm}}{p_f}\frac{A_t(3)}{A_t} = \frac{3\,p_{atm}}{\gamma(h-0.6) + p_{atm}}$$

so that

$$h^2 + \left(\frac{p_{atm}}{\gamma} - 0.6\right)h - 3\frac{p_{atm}}{\gamma} = 0$$

For $\gamma = 9.80 \frac{kN}{m^3}$ and $p_{atm} = 101\,kPa$,

$$h^2 + \left(\frac{101\,kPa}{9.80\frac{kN}{m^3}} - 0.6\,m\right)h - \frac{3(101\,kPa)}{9.80\frac{kN}{m^3}} = 0$$

or

$$h^2 + 9.71\,h - 30.9 = 0$$

so that

$$h = \frac{-9.71 \pm \sqrt{(9.71)^2 + 4(30.9)}}{2} = 2.53\,m$$

Thus, from Eq. (1)

$$p_f\,(gage) = \left(9.80\frac{kN}{m^3}\right)(2.53\,m - 0.6\,m) = \underline{\underline{18.9\,kPa}}$$

(con't)

2.90 (con't)

(b) For equilibrium of tank (see free-body-diagram),

$$T = p_f A_t - \mathcal{W}_t$$

where $\mathcal{W}_t$ ~ tank weight, and for steel block

$$T = \mathcal{W}_s - F_{B_s} = \mathcal{V}_s\,(\gamma_s - \gamma)$$

Thus,

$$\mathcal{V}_s = \frac{T}{\gamma_s - \gamma} = \frac{p_f A_t - \mathcal{W}_t}{\gamma_s - \gamma}$$

$$= \frac{\left(18.9 \times 10^3 \frac{N}{m^2}\right)\left(\frac{\pi}{4}\right)(1\,m)^2 - (90\,kg)\left(9.81\,\frac{m}{s^2}\right)}{\left(7.840 \times 10^3 \frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right) - 9.80 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{0.208\ m^3}}$$